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Matrices and Arrays

You can watch the Getting Started with MATLAB video demo for an overview of the major functionality.

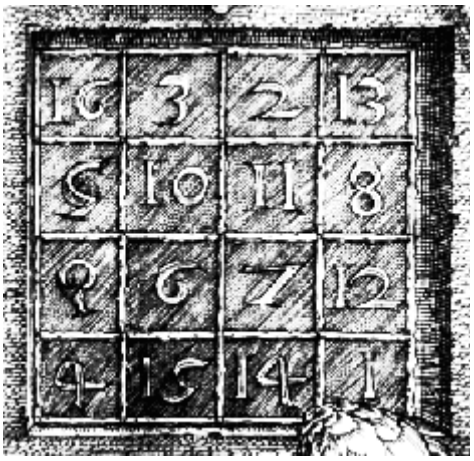
Matrices and Magic Squares

In this section...

About Matrices

In the MATLAB environment, a matrix is a rectangular array of numbers. Special meaning is sometimes attached to 1-by-1 matrices, which are scalars, and to matrices with only one row or column, which are vectors. MATLAB has other ways of storing both numeric and nonnumeric data, but in the beginning, it is usually best to think of everything as a matrix. The operations in MATLAB are designed to be as natural as possible. Where other programming languages work with numbers one at a time, MATLAB allows you to work with entire matrices quickly and easily. A good example matrix, used throughout this book, appears in the Renaissance engraving Melencolia I by the German artist and amateur mathematician Albrecht Dürer.

This image is filled with mathematical symbolism, and if you look carefully, you will see a matrix in the upper right corner. This matrix is known as a magic square and was believed by many in Dürer's time to have genuinely magical properties. It does turn out to have some fascinating characteristics worth exploring.



Entering Matrices

The best way for you to get started with MATLAB is to learn how to handle matrices. Start MATLAB and follow along with each example.

You can enter matrices into MATLAB in several different ways:

- Enter an explicit list of elements.
- Load matrices from external data files.
- Generate matrices using built-in functions.
- Create matrices with your own functions and save them in files.

Start by entering Dürer's matrix as a list of its elements. You only have to follow a few basic conventions:

- Separate the elements of a row with blanks or commas.
- Use a semicolon, `;` , to indicate the end of each row.
- Surround the entire list of elements with square brackets, `[ ]`.

To enter Dürer's matrix, simply type in the Command Window

```
A = [16 3 2 13; 5 10 11 8; 9 6 7 12; 4 15 14 1]
```

MATLAB displays the matrix you just entered:

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

This matrix matches the numbers in the engraving. Once you have entered the matrix, it is automatically remembered in the MATLAB workspace. You can refer to it simply as `A`. Now that you have `A` in the workspace, take a look at what makes it so interesting. Why is it magic?

sum, transpose, and diag

You are probably already aware that the special properties of a magic square have to do with the various ways of summing its elements. If you take the sum along any row or column, or along either of the two main diagonals, you will always get the same number. Let us verify that using MATLAB. The first statement to try is

```
sum(A)
```

MATLAB replies with

```
ans =
    34    34    34    34
```

When you do not specify an output variable, MATLAB uses the variable `ans`, short for *answer*, to store the results of a calculation. You have computed a row vector containing the sums of the columns of `A`. Each of the columns has the same sum, the *magic* sum, 34.

How about the row sums? MATLAB has a preference for working with the columns of a matrix, so one way to get the row sums is to transpose the matrix, compute the column sums of the transpose, and then transpose the result. For an additional way that avoids the double transpose use the dimension argument for the `sum` function.

MATLAB has two transpose operators. The apostrophe operator (e.g., `A'`) performs a complex conjugate transposition. It flips a matrix about its main

diagonal, and also changes the sign of the imaginary component of any complex elements of the matrix. The dot-apostrophe operator (e.g., `A.'`), transposes without affecting the sign of complex elements. For matrices containing all real elements, the two operators return the same result.

So

```
A'
```

produces

```
ans =
    16     5     9     4
     3    10     6    15
     2    11     7    14
    13     8    12     1
```

and

```
sum(A')'
```

produces a column vector containing the row sums

```
ans =
    34
    34
    34
    34
```

The sum of the elements on the main diagonal is obtained with the `sum` and the `diag` functions:

```
diag(A)
```

produces

```
ans =
    16
    10
     7
     1
```

and

```
sum(diag(A))
```

produces

```
ans =
    34
```

The other diagonal, the so-called *antidiagonal,* is not so important mathematically, so MATLAB does not have a ready-made function for it. But a function originally intended for use in graphics, `fliplr`, flips a matrix from left to right:

```
sum(diag(fliplr(A)))
ans =
    34
```

You have verified that the matrix in Dürer's engraving is indeed a magic square and, in the process, have sampled a few MATLAB matrix operations. The following sections continue to use this matrix to illustrate additional MATLAB capabilities.

Subscripts

The element in row `i` and column `j` of `A` is denoted by `A(i,j)`. For example, `A(4,2)` is the number in the fourth row and second column. For the magic square, `A(4,2)` is `15`. So to compute the sum of the elements in the fourth column of `A`, type

```
A(1,4) + A(2,4) + A(3,4) + A(4,4)
```

This subscript produces

```
ans =
     34
```

but is not the most elegant way of summing a single column.

It is also possible to refer to the elements of a matrix with a single subscript, `A(k)`. A single subscript is the usual way of referencing row and column vectors. However, it can also apply to a fully two-dimensional matrix, in

which case the array is regarded as one long column vector formed from the columns of the original matrix. So, for the magic square, `A(8)` is another way of referring to the value `15` stored in `A(4,2)`.

If you try to use the value of an element outside of the matrix, it is an error:

```
t = A(4,5)
Index exceeds matrix dimensions.
```

Conversely, if you store a value in an element outside of the matrix, the size increases to accommodate the newcomer:

```
X = A;
X(4,5) = 17

X =
    16     3     2    13     0
     5    10    11     8     0
     9     6     7    12     0
     4    15    14     1    17
```

The Colon Operator

The colon, `:`, is one of the most important MATLAB operators. It occurs in several different forms. The expression

```
1:10
```

is a row vector containing the integers from 1 to 10:

```
1     2     3     4     5     6     7     8     9    10
```

To obtain nonunit spacing, specify an increment. For example,

```
100:-7:50
```

is

```
100    93    86    79    72    65    58    51
```

and

```
0:pi/4:pi
```

is

```
0    0.7854    1.5708    2.3562    3.1416
```

Subscript expressions involving colons refer to portions of a matrix:

```
A(1:k,j)
```

is the first `k` elements of the `j`th column of `A`. Thus:

```
sum(A(1:4,4))
```

computes the sum of the fourth column. However, there is a better way to perform this computation. The colon by itself refers to *all* the elements in a row or column of a matrix and the keyword `end` refers to the *last* row or column. Thus:

```
sum(A(:,end))
```

computes the sum of the elements in the last column of `A`:

```
ans =
     34
```

Why is the magic sum for a 4-by-4 square equal to 34? If the integers from 1 to 16 are sorted into four groups with equal sums, that sum must be

```
sum(1:16)/4
```

which, of course, is

```
ans =
     34
```

The magic Function

MATLAB actually has a built-in function that creates magic squares of almost any size. Not surprisingly, this function is named `magic`:

```
B = magic(4)
B =
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
```

This matrix is almost the same as the one in the Dürer engraving and has all the same "magic" properties; the only difference is that the two middle columns are exchanged.

To make this `B` into Dürer's `A`, swap the two middle columns:

```
A = B(:,[1 3 2 4])
```

This subscript indicates that—for each of the rows of matrix `B`—reorder the elements in the order 1, 3, 2, 4. It produces:

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

Expressions

In this section...

Variables

Like most other programming languages, the MATLAB language provides mathematical *expressions*, but unlike most programming languages, these expressions involve entire matrices.

MATLAB does not require any type declarations or dimension statements. When MATLAB encounters a new variable name, it automatically creates the variable and allocates the appropriate amount of storage. If the variable already exists, MATLAB changes its contents and, if necessary, allocates new storage. For example,

```
num_students = 25
```

creates a 1-by-1 matrix named `num_students` and stores the value 25 in its single element. To view the matrix assigned to any variable, simply enter the variable name.

Variable names consist of a letter, followed by any number of letters, digits, or underscores. MATLAB is case sensitive; it distinguishes between uppercase and lowercase letters. `A` and `a` are *not* the same variable.

Although variable names can be of any length, MATLAB uses only the first `N` characters of the name, (where `N` is the number returned by the function `namelengthmax`), and ignores the rest. Hence, it is important to make each variable name unique in the first `N` characters to enable MATLAB to distinguish variables.

```
N = namelengthmax
N =
    63
```

The genvarname function can be useful in creating variable names that are both valid and unique.

Numbers

MATLAB uses conventional decimal notation, with an optional decimal point and leading plus or minus sign, for numbers. Scientific notation uses the letter e to specify a power-of-ten scale factor. Imaginary numbers use either i or j as a suffix. Some examples of legal numbers are

```
3              -99            0.0001
9.6397238      1.60210e-20    6.02252e23
1i             -3.14159j      3e5i
```

MATLAB stores all numbers internally using the *long* format specified by the IEEE® floating-point standard. Floating-point numbers have a finite *precision* of roughly 16 significant decimal digits and a finite *range* of roughly 10^{-308} to 10^{+308}.

Numbers represented in the double format have a maximum precision of 52 bits. Any double requiring more bits than 52 loses some precision. For example, the following code shows two unequal values to be equal because they are both truncated:

```
x = 36028797018963968;
y = 36028797018963972;
x == y
ans =
      1
```

Integers have available precisions of 8-bit, 16-bit, 32-bit, and 64-bit. Storing the same numbers as 64-bit integers preserves precision:

```
x = uint64(36028797018963968);
y = uint64(36028797018963972);
x == y
ans =
```

```
     0
```

The section "Avoiding Common Problems with Floating-Point Arithmetic" gives a few of the examples showing how IEEE floating-point arithmetic affects computations in MATLAB. For more examples and information, see Technical Note 1108 — Common Problems with Floating-Point Arithmetic.

MATLAB software stores the real and imaginary parts of a complex number. It handles the magnitude of the parts in different ways depending on the context. For instance, the `sort` function sorts based on magnitude and resolves ties by phase angle.

```
sort([3+4i, 4+3i])
ans =
   4.0000 + 3.0000i   3.0000 + 4.0000i
```

This is because of the phase angle:

```
angle(3+4i)
ans =
    0.9273
angle(4+3i)
ans =
    0.6435
```

The "equal to" relational operator == requires both the real and imaginary parts to be equal. The other binary relational operators > <, >=, and <= ignore the imaginary part of the number and consider the real part only.

Operators

Expressions use familiar arithmetic operators and precedence rules.

+	Addition
-	Subtraction
*	Multiplication
/	Division

\	Left division (described in "Linear Algebra" in the MATLAB documentation)
^	Power
'	Complex conjugate transpose
( )	Specify evaluation order

Functions

MATLAB provides a large number of standard elementary mathematical functions, including `abs`, `sqrt`, `exp`, and `sin`. Taking the square root or logarithm of a negative number is not an error; the appropriate complex result is produced automatically. MATLAB also provides many more advanced mathematical functions, including Bessel and gamma functions. Most of these functions accept complex arguments. For a list of the elementary mathematical functions, type

```
help elfun
```

For a list of more advanced mathematical and matrix functions, type

```
help specfun
help elmat
```

Some of the functions, like `sqrt` and `sin`, are *built in*. Built-in functions are part of the MATLAB core so they are very efficient, but the computational details are not readily accessible. Other functions are implemented in the MATLAB programing language, so their computational details are accessible.

There are some differences between built-in functions and other functions. For example, for built-in functions, you cannot see the code. For other functions, you can see the code and even modify it if you want.

Several special functions provide values of useful constants.

`pi`	3.14159265...
`i`	Imaginary unit, $\sqrt{-1}$
`j`	Same as `i`

`eps`	Floating-point relative precision, $\varepsilon = 2^{-52}$
`realmin`	Smallest floating-point number, 2^{-1022}
`realmax`	Largest floating-point number, $(2-\varepsilon)2^{1023}$
`Inf`	Infinity
`NaN`	Not-a-number

Infinity is generated by dividing a nonzero value by zero, or by evaluating well defined mathematical expressions that *overflow*, i.e., exceed `realmax`. Not-a-number is generated by trying to evaluate expressions like `0/0` or `Inf-Inf` that do not have well defined mathematical values.

The function names are not reserved. It is possible to overwrite any of them with a new variable, such as

```
eps = 1.e-6
```

and then use that value in subsequent calculations. The original function can be restored with

```
clear eps
```

Examples of Expressions

You have already seen several examples of MATLAB expressions. Here are a few more examples, and the resulting values:

```
rho = (1+sqrt(5))/2
rho =
    1.6180

a = abs(3+4i)
a =
     5

z = sqrt(besselk(4/3,rho-i))
z =
   0.3730+ 0.3214i
```

```
huge = exp(log(realmax))
huge =
  1.7977e+308

toobig = pi*huge
toobig =
   Inf
```

Working with Matrices

In this section...

Generating Matrices

MATLAB software provides four functions that generate basic matrices.

`zeros`	All zeros
`ones`	All ones
`rand`	Uniformly distributed random elements
`randn`	Normally distributed random elements

Here are some examples:

```
Z = zeros(2,4)
Z =
     0     0     0     0
     0     0     0     0

F = 5*ones(3,3)
F =
     5     5     5
     5     5     5
     5     5     5

N = fix(10*rand(1,10))
N =
     9     2     6     4     8     7     4     0     8     4
```

```
R = randn(4,4)
R =
    0.6353    0.0860   -0.3210   -1.2316
   -0.6014   -2.0046    1.2366    1.0556
    0.5512   -0.4931   -0.6313   -0.1132
   -1.0998    0.4620   -2.3252    0.3792
```

The load Function

The `load` function reads binary files containing matrices generated by earlier MATLAB sessions, or reads text files containing numeric data. The text file should be organized as a rectangular table of numbers, separated by blanks, with one row per line, and an equal number of elements in each row. For example, outside of MATLAB, create a text file containing these four lines:

```
16.0     3.0     2.0    13.0
 5.0    10.0    11.0     8.0
 9.0     6.0     7.0    12.0
 4.0    15.0    14.0     1.0
```

Save the file as `magik.dat` in the current directory. The statement

```
load magik.dat
```

reads the file and creates a variable, `magik`, containing the example matrix.

An easy way to read data into MATLAB from many text or binary formats is to use the Import Wizard.

Saving Code to a File

You can create matrices using text files containing MATLAB code. Use the MATLAB Editor or another text editor to create a file containing the same statements you would type at the MATLAB command line. Save the file under a name that ends in `.m`.

For example, create a file in the current directory named `magik.m` containing these five lines:

```
A = [16.0     3.0     2.0    13.0
      5.0    10.0    11.0     8.0
      9.0     6.0     7.0    12.0
      4.0    15.0    14.0     1.0 ];
```

The statement

```
magik
```

reads the file and creates a variable, `A`, containing the example matrix.

Concatenation

Concatenation is the process of joining small matrices to make bigger ones. In fact, you made your first matrix by concatenating its individual elements. The pair of square brackets, `[]`, is the concatenation operator. For an example, start with the 4-by-4 magic square, `A`, and form

```
B = [A  A+32; A+48  A+16]
```

The result is an 8-by-8 matrix, obtained by joining the four submatrices:

```
B =

    16     3     2    13    48    35    34    45
     5    10    11     8    37    42    43    40
     9     6     7    12    41    38    39    44
     4    15    14     1    36    47    46    33
    64    51    50    61    32    19    18    29
    53    58    59    56    21    26    27    24
    57    54    55    60    25    22    23    28
    52    63    62    49    20    31    30    17
```

This matrix is halfway to being another magic square. Its elements are a rearrangement of the integers `1:64`. Its column sums are the correct value for an 8-by-8 magic square:

```
sum(B)

ans =
   260   260   260   260   260   260   260   260
```

But its row sums, `sum(B')'`, are not all the same. Further manipulation is necessary to make this a valid 8-by-8 magic square.

Deleting Rows and Columns

You can delete rows and columns from a matrix using just a pair of square brackets. Start with

```
X = A;
```

Then, to delete the second column of `X`, use

```
X(:,2) = []
```

This changes `X` to

```
X =
    16     2    13
     5    11     8
     9     7    12
     4    14     1
```

If you delete a single element from a matrix, the result is not a matrix anymore. So, expressions like

```
X(1,2) = []
```

result in an error. However, using a single subscript deletes a single element, or sequence of elements, and reshapes the remaining elements into a row vector. So

```
X(2:2:10) = []
```

results in

```
X =
    16     9     2     7    13    12     1
```

More About Matrices and Arrays

In this section...

Linear Algebra

Informally, the terms *matrix* and *array* are often used interchangeably. More precisely, a *matrix* is a two-dimensional numeric array that represents a linear transformation. The mathematical operations defined on matrices are the subject of linear algebra.

Dürer's magic square

```
A = [16     3     2    13
      5    10    11     8
      9     6     7    12
      4    15    14     1 ]
```

provides several examples that give a taste of MATLAB matrix operations. You have already seen the matrix transpose, `A'`. Adding a matrix to its transpose produces a *symmetric* matrix:

```
A + A'

ans =
    32     8    11    17
     8    20    17    23
    11    17    14    26
    17    23    26     2
```

The multiplication symbol, `*`, denotes the *matrix* multiplication involving inner products between rows and columns. Multiplying the transpose of a matrix by the original matrix also produces a symmetric matrix:

```
A'*A

ans =
   378   212   206   360
   212   370   368   206
   206   368   370   212
   360   206   212   378
```

The determinant of this particular matrix happens to be zero, indicating that the matrix is *singular*:

```
d = det(A)

d =
     0
```

The reduced row echelon form of `A` is not the identity:

```
R = rref(A)

R =
     1     0     0     1
     0     1     0    -3
     0     0     1     3
     0     0     0     0
```

Because the matrix is singular, it does not have an inverse. If you try to compute the inverse with

```
X = inv(A)
```

you will get a warning message:

```
Warning: Matrix is close to singular or badly scaled.
         Results may be inaccurate. RCOND = 9.796086e-018.
```

Roundoff error has prevented the matrix inversion algorithm from detecting exact singularity. But the value of `rcond`, which stands for *reciprocal*

condition estimate, is on the order of `eps`, the floating-point relative precision, so the computed inverse is unlikely to be of much use.

The eigenvalues of the magic square are interesting:

```
e = eig(A)

e =
   34.0000
    8.0000
    0.0000
   -8.0000
```

One of the eigenvalues is zero, which is another consequence of singularity. The largest eigenvalue is 34, the magic sum. That sum results because the vector of all ones is an eigenvector:

```
v = ones(4,1)

v =
     1
     1
     1
     1

A*v

ans =
    34
    34
    34
    34
```

When a magic square is scaled by its magic sum,

```
P = A/34
```

the result is a *doubly stochastic* matrix whose row and column sums are all 1:

```
P =
    0.4706    0.0882    0.0588    0.3824
    0.1471    0.2941    0.3235    0.2353
    0.2647    0.1765    0.2059    0.3529
    0.1176    0.4412    0.4118    0.0294
```

Such matrices represent the transition probabilities in a *Markov process*. Repeated powers of the matrix represent repeated steps of the process. For this example, the fifth power

```
P^5
```

is

```
    0.2507    0.2495    0.2494    0.2504
    0.2497    0.2501    0.2502    0.2500
    0.2500    0.2498    0.2499    0.2503
    0.2496    0.2506    0.2505    0.2493
```

This shows that as k approaches infinity, all the elements in the kth power, p^k, approach $1/4$.

Finally, the coefficients in the characteristic polynomial

```
poly(A)
```

are

```
     1   -34   -64  2176     0
```

These coefficients indicate that the characteristic polynomial

$$\det(A - \lambda I)$$

is

$$\lambda^4 - 34\lambda^3 - 64\lambda^2 + 2176\lambda$$

The constant term is zero because the matrix is singular. The coefficient of the cubic term is -34 because the matrix is magic!

Arrays

When they are taken away from the world of linear algebra, matrices become two-dimensional numeric arrays. Arithmetic operations on arrays are done element by element. This means that addition and subtraction are the same for arrays and matrices, but that multiplicative operations are different. MATLAB uses a dot, or decimal point, as part of the notation for multiplicative array operations.

The list of operators includes

`+`	Addition
`-`	Subtraction
`.*`	Element-by-element multiplication
`./`	Element-by-element division
`.\`	Element-by-element left division
`.^`	Element-by-element power
`.'`	Unconjugated array transpose

If the Dürer magic square is multiplied by itself with array multiplication

```
A.*A
```

the result is an array containing the squares of the integers from 1 to 16, in an unusual order:

```
ans =
   256     9     4   169
    25   100   121    64
    81    36    49   144
    16   225   196     1
```

Building Tables

Array operations are useful for building tables. Suppose `n` is the column vector

```
n = (0:9)';
```

Then

```
pows = [n  n.^2  2.^n]
```

builds a table of squares and powers of 2:

```
pows =
     0     0     1
     1     1     2
     2     4     4
     3     9     8
     4    16    16
     5    25    32
     6    36    64
     7    49   128
     8    64   256
     9    81   512
```

The elementary math functions operate on arrays element by element. So

```
format short g
x = (1:0.1:2)';
logs = [x log10(x)]
```

builds a table of logarithms.

```
 logs =
      1.0           0
      1.1     0.04139
      1.2     0.07918
      1.3     0.11394
      1.4     0.14613
      1.5     0.17609
      1.6     0.20412
      1.7     0.23045
      1.8     0.25527
      1.9     0.27875
      2.0     0.30103
```

Multivariate Data

MATLAB uses column-oriented analysis for multivariate statistical data. Each column in a data set represents a variable and each row an observation. The `(i,j)`th element is the `i`th observation of the `j`th variable.

As an example, consider a data set with three variables:

- Heart rate
- Weight
- Hours of exercise per week

For five observations, the resulting array might look like

```
D = [ 72          134          3.2
      81          201          3.5
      69          156          7.1
      82          148          2.4
      75          170          1.2 ]
```

The first row contains the heart rate, weight, and exercise hours for patient 1, the second row contains the data for patient 2, and so on. Now you can apply many MATLAB data analysis functions to this data set. For example, to obtain the mean and standard deviation of each column, use

```
mu = mean(D), sigma = std(D)

mu =
      75.8       161.8         3.48

sigma =
    5.6303      25.499       2.2107
```

For a list of the data analysis functions available in MATLAB, type

```
help datafun
```

If you have access to the Statistics Toolbox™ software, type

```
help stats
```

Scalar Expansion

Matrices and scalars can be combined in several different ways. For example, a scalar is subtracted from a matrix by subtracting it from each element. The average value of the elements in our magic square is 8.5, so

```
B = A - 8.5
```

forms a matrix whose column sums are zero:

```
B =
      7.5     -5.5     -6.5      4.5
     -3.5      1.5      2.5     -0.5
      0.5     -2.5     -1.5      3.5
     -4.5      6.5      5.5     -7.5

sum(B)

ans =
     0     0     0     0
```

With scalar expansion, MATLAB assigns a specified scalar to all indices in a range. For example,

```
B(1:2,2:3) = 0
```

zeroes out a portion of `B`:

```
B =
      7.5        0        0      4.5
     -3.5        0        0     -0.5
      0.5     -2.5     -1.5      3.5
     -4.5      6.5      5.5     -7.5
```

Logical Subscripting

The logical vectors created from logical and relational operations can be used to reference subarrays. Suppose `X` is an ordinary matrix and `L` is a matrix of the same size that is the result of some logical operation. Then `X(L)` specifies the elements of `X` where the elements of `L` are nonzero.

This kind of subscripting can be done in one step by specifying the logical operation as the subscripting expression. Suppose you have the following set of data:

```
x = [2.1 1.7 1.6 1.5 NaN 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8];
```

The `NaN` is a marker for a missing observation, such as a failure to respond to an item on a questionnaire. To remove the missing data with logical indexing, use `isfinite(x)`, which is true for all finite numerical values and false for `NaN` and `Inf`:

```
x = x(isfinite(x))
x =
  2.1 1.7 1.6 1.5 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8
```

Now there is one observation, `5.1`, which seems to be very different from the others. It is an *outlier*. The following statement removes outliers, in this case those elements more than three standard deviations from the mean:

```
x = x(abs(x-mean(x)) <= 3*std(x))
x =
  2.1 1.7 1.6 1.5 1.9 1.8 1.5 1.8 1.4 2.2 1.6 1.8
```

For another example, highlight the location of the prime numbers in Dürer's magic square by using logical indexing and scalar expansion to set the nonprimes to 0. (See "The magic Function" on page 2-9.)

```
A(~isprime(A)) = 0

A =
     0     3     2    13
     5     0    11     0
     0     0     7     0
     0     0     0     0
```

The find Function

The `find` function determines the indices of array elements that meet a given logical condition. In its simplest form, `find` returns a column vector of indices. Transpose that vector to obtain a row vector of indices. For example, start again with Dürer's magic square. (See "The magic Function" on page 2-9.)

```
k = find(isprime(A))'
```

picks out the locations, using one-dimensional indexing, of the primes in the magic square:

```
k =
     2     5     9    10    11    13
```

Display those primes, as a row vector in the order determined by k, with

```
A(k)

ans =
     5     3     2    11     7    13
```

When you use k as a left-hand-side index in an assignment statement, the matrix structure is preserved:

```
A(k) = NaN

A =
    16   NaN   NaN   NaN
   NaN    10   NaN     8
     9     6   NaN    12
     4    15    14     1
```

Controlling Command Window Input and Output

In this section...

The format Function

The `format` function controls the numeric format of the values displayed. The function affects only how numbers are displayed, not how MATLAB software computes or saves them. Here are the different formats, together with the resulting output produced from a vector `x` with components of different magnitudes.

Note To ensure proper spacing, use a fixed-width font, such as Courier.

```
x = [4/3 1.2345e-6]

format short

   1.3333    0.0000

format short e

   1.3333e+000  1.2345e-006

format short g

   1.3333  1.2345e-006

format long

   1.33333333333333   0.00000123450000
```

```
format long e

   1.333333333333333e+000    1.234500000000000e-006

format long g

   1.33333333333333                1.2345e-006

format bank

   1.33          0.00

format rat

   4/3          1/810045

format hex

   3ff5555555555555   3eb4b6231abfd271
```

If the largest element of a matrix is larger than 10^3 or smaller than 10^{-3}, MATLAB applies a common scale factor for the short and long formats.

In addition to the `format` functions shown above

```
format compact
```

suppresses many of the blank lines that appear in the output. This lets you view more information on a screen or window. If you want more control over the output format, use the `sprintf` and `fprintf` functions.

Suppressing Output

If you simply type a statement and press **Return** or **Enter**, MATLAB automatically displays the results on screen. However, if you end the line with a semicolon, MATLAB performs the computation but does not display any output. This is particularly useful when you generate large matrices. For example,

```
A = magic(100);
```

Entering Long Statements

If a statement does not fit on one line, use an ellipsis (three periods), `...`, followed by **Return** or **Enter** to indicate that the statement continues on the next line. For example,

```
s = 1 -1/2 + 1/3 -1/4 + 1/5 - 1/6 + 1/7 ...
      - 1/8 + 1/9 - 1/10 + 1/11 - 1/12;
```

Blank spaces around the `=`, `+`, and `-` signs are optional, but they improve readability.

Command Line Editing

Various arrow and control keys on your keyboard allow you to recall, edit, and reuse statements you have typed earlier. For example, suppose you mistakenly enter

```
rho = (1 + sqt(5))/2
```

You have misspelled `sqrt`. MATLAB responds with

```
Undefined function 'sqt' for input arguments of type 'double'.
```

Instead of retyping the entire line, simply press the ↑ key. The statement you typed is redisplayed. Use the ← key to move the cursor over and insert the missing `r`. Repeated use of the ↑ key recalls earlier lines. Typing a few characters and then the ↑ key finds a previous line that begins with those characters. You can also copy previously executed statements from the Command History. For more information, see “Command History” on page 7-6.